resonance is significant in the photon energy region of this experiment and thus possibly help to resolve the ambiguity in the interpretation of our result. A measure of the contribution that the S-wave amplitude makes in the region of the second resonance might also be obtained.

In conclusion, under the assumption that the amplitudes for the production of the intermediate states from protons and neutrons are the same, the result of our measurement of the polarization of the proton from the $(n,p\pi^-)$ reaction has been interpreted as indicating that the interference between the first and second resonances may not be the dominant contribution to the polarization for photon energies in the neighborhood of 715 MeV. Significant contributions from either the interference between the first resonance and the possible new resonance suggested by the π, p scattering measurements results, or between the second resonance and the third resonance, or a combination of these two possibilities seem to be required at this energy. These possibilities do not seem to be in disagreement with the $(p,p\pi^0)$ polarization measurements. We are not able, however, to distinguish between these alternatives.

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Low-Energy Photoproduction of Λ^0 and K^+ from Protons*

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A model is constructed for low-energy $\gamma+p \rightarrow K^+ + \Lambda^0$ reactions in accordance with dispersion theory by neglecting faraway singularities. Thus, besides the Born terms due to one-nucleon intermediate state and K^+ exchange, we also employ the K^* exchange and a resonance in the final state similar to that found in the reaction $\pi^-+p \rightarrow K^0+\Lambda^0$. A fairly good fit with recently measured data is obtained. [See R. L. Anderson *et al.*, Phys. Rev. Letters 9, 131 (1962).] The choice of parameters is briefly discussed.

I. INTRODUCTION

New experiments on the photoproduction of Λ^0 and K^+ from protons have recently been completed by a Cornell University group.[1] The results they obtained differ considerably from the old data.[2] The gross features of the newly measured differential cross sections in the center-of-mass system are:

(a) The K^+ meson tends to peak forward with respect to the incident photon.

(b) The angular distribution is of the form $a+b\cos\theta+c\cos^2\theta$.

(c) The excitation curve $(d\sigma/d\Omega)_{\theta=\pi/2}$ has an S-wave rise near threshold. It seems to reach a maximum around incident photon energy $E_\gamma \cong 1060$ MeV.

This "simplicity" of the existing data offers a striking contrast to its theoretical interpretations. We know that the Watson[3] theorem, which is extremely important in pion photoproduction from nucleons, cannot be applied here. This is because already in the energy range in which experimental data are available, there are many open channels: γN, $(n\pi)N$,[4] ΛK, as well as ΣK. An approach which uses dispersion integrals for partial-wave amplitudes, as has been done to many reactions, would lead to a very complicated set of coupled integral equations, and there seems to be little hope of solving them.

A number of authors[5] have discussed the possibility of applying to this problem the Cini-Fubini approximation[6] to the Mandelstam representation. They considered the contribution of the perturbation Born terms, the πN resonances, the various pion-hyperon resonances, and the $K\pi$ resonance. That no higher powers in $\cos\theta$ than 2 are required to describe the angular distribution of the K^+ meson, however, suggests the possibility of a low-energy approximation. In this

* Supported in part by the Office of Naval Research.

[1] R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. J. Sadoff, Phys. Rev. Letters 9, 131 (1962).

[2] A summary of old data can be found in the article by F. Turkot, in *Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1960), p. 369.

[3] K. M. Watson, Phys. Rev. 95, 228 (1954).

[4] $n=1, 2, 3, 4, 5$. Although we may neglect $n>3$, $2\pi N$ seems definitely important.

[5] M. Gourdin, Nuovo Cimento 20, 1035 (1961); S. Hatsukade and H. J. Schnitzer, Phys. Rev. 128, 468 (1962); Dufour and M. Gourdin (to be published).

[6] M. Cini and Fubini, Ann. Phys. (N. Y.) 10, 352 (1960).

approximation we shall neglect high-angular-momentum multipoles, in particular, the $F_{5/2}$ third πN resonance. After briefly discussing our notation and kinematics in Sec. II, we present arguments in Sec. III for neglecting more terms in the low-energy region. In Sec. IV we write down the amplitude for the production process. It will contain a number of parameters. Section V will then be devoted to comparison with experimental data. A discussion is then given in Sec. VI. In the Appendix we try to relate the ratio C_1/C_2, which will be defined in Sec. IV, to various coupling constants involving the K meson.

II. KINEMATICS

We define the four momentum variables as in Fig. 1.[7] The invariant "Mandelstam variables" are given by[8]

$$\begin{aligned} s&=(k+p_1)^2, \quad \text{Channel I;}\\ t&=(q-k)^2, \quad \text{Channel II;}\\ u&=(p_1-q)^2, \quad \text{Channel III.}\end{aligned} \tag{1}$$

They satisfy

$$s+t+u=m_N^2+m_\Lambda^2+m_K^2.$$

The T matrix for the production process is related to the S matrix by the following:

$$S_{fi}=-\frac{i}{(2\pi)^2}\delta^4(k+p_1-q-p_2)\left(\frac{m_\Lambda m_N}{4E_1E_2k\omega}\right)^{1/2}\times\bar{u}(p_2)T_{fi}u(p_1), \tag{2}$$

where $E_1\equiv(|\mathbf{p}_1|^2+m_N^2)^{1/2}$, $E_2\equiv(|\mathbf{p}_2|^2+m_\Lambda^2)^{1/2}$, $k=k_0$, $\omega=(|\mathbf{q}|^2+m_K^2)^{1/2}$ are the energies of the nucleon, the hyperon, the photon, and the K^+ meson.

We also define, in the barycentric system:

$$\begin{aligned} k&=(|\mathbf{k}|,\mathbf{k}), \quad p_1=(E_1,-\mathbf{k}),\\ q&=(\omega,\mathbf{q}), \quad p_2=(E_2,-\mathbf{q}).\end{aligned} \tag{3}$$

Thus if W denotes the total energy in Channel I, then we have

$$\begin{aligned} s&=W^2,\\ t&=m_K^2-2\omega k+2kq\cos\theta,\\ u&=m_N^2+m_K^2-2E_1\omega-2kq\cos\theta,\end{aligned} \tag{4}$$

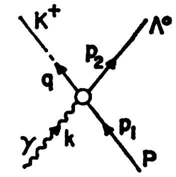

FIG. 1. Photoproduction kinematics.

θ being the angle between K^+ and the incident photon in the c.m. system. Several useful relations are

$$\begin{aligned} k^2&=\frac{(s-m_N^2)^2}{4s}, \quad q^2=\frac{[s-(m_\Lambda+m_K)^2][s-(m_\Lambda-m_K)^2]}{4s},\\ \omega&=\frac{s-(m_\Lambda^2-m_K^2)}{2W}, \quad E_1=\frac{s+m_N^2}{2W},\\ E_2&=\frac{s+(m_\Lambda^2-m_K^2)}{2W}.\end{aligned} \tag{5}$$

Since it is established that the parity $P(K\Lambda)=-1$,[9] we may write[7]

$$T_{fi}=\sum_{i=1}^{4}A_i\mathfrak{M}_i, \tag{6}$$

$$\begin{aligned} \mathfrak{M}_1&=-\gamma_5\gamma\cdot\epsilon\gamma\cdot k,\\ \mathfrak{M}_2&=2\gamma_5(P\cdot\epsilon q\cdot k-P\cdot kq\cdot\epsilon),\\ \mathfrak{M}_3&=-\gamma_5(q\cdot k\gamma\cdot\epsilon-\gamma\cdot kq\cdot\epsilon),\\ \mathfrak{M}_4&=-2\gamma_5(\gamma\cdot\epsilon P\cdot k-\gamma\cdot kP\cdot\epsilon-m_N\gamma\cdot\epsilon\gamma\cdot k),\end{aligned} \tag{7}$$

where $P=\frac{1}{2}(p_1+p_2)$, ϵ is the polarization 4-vector of the photon; the metric used is defined in footnote 8. In order to analyze the differential cross section in terms of multipoles, it is convenient to write[7]

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}|\chi_f^+\mathfrak{F}\chi_i|^2, \tag{8}$$

so that

$$\chi_f^+\mathfrak{F}\chi_i=\frac{(m_Nm_\Lambda)^{1/2}}{4\pi W}\bar{u}(p_2)T_{fi}u(p_1). \tag{9}$$

$\mathfrak{F}$, in turn, is written as

$$\mathfrak{F}=i\boldsymbol{\sigma}\cdot\hat{\epsilon}\mathfrak{F}_1+\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot\hat{k}\times\hat{\epsilon}\mathfrak{F}_2+i\boldsymbol{\sigma}\cdot\hat{k}\hat{q}\cdot\hat{\epsilon}\mathfrak{F}_3+i\boldsymbol{\sigma}\cdot\hat{q}\hat{q}\cdot\hat{\epsilon}\mathfrak{F}_4. \tag{10}$$

[7] Kinematics for photoproduction processes has been discussed by many authors: G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957); Fayyazuddin, *ibid.* **123**, 1882 (1961); J. S. Ball, *ibid.* **124**, 2014 (1961).

[8] The metric used is $g_{00}=+1$, $g_{ii}=-1$, $i=1, 2, 3$; thus $a\cdot b=a_0b_0-\mathbf{a}\cdot\mathbf{b}$. The γ matrices are defined so that $\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$, $\gamma_0^*=\gamma_0$, $\gamma_i^*=-\gamma_i$, $i=1, 2, 3$.

[9] M. M. Block, F. Anderson, A. Pevsner, E. Harth, J. Leitner, and H. Cohn, Phys. Rev. Letters **3**, 291 (1959).

The decomposition into multipoles is

$$\mathfrak{F}_1=\sum_{l=0}^{\infty}[lM_{l^+}+E_{l^+}]P_{l+1}'(\cos\theta)+\sum_{l=1}^{\infty}[(l+1)M_{l^-}+E_{l^-}]P_{l-1}'(\cos\theta),$$

$$\mathfrak{F}_2=\sum_{l=1}^{\infty}[(l+1)M_{l^-}+lM_{l^-}]P_l'(\cos\theta), \tag{11}$$

$$\mathfrak{F}_3=\sum_{l=1}^{\infty}[E_{l^+}-M_{l^+}]P_{l+1}''(\cos\theta)+[E_{l^-}+M_{l^-}]P_{l-1}''(\cos\theta),$$

$$\mathfrak{F}_4=\sum_{l=1}^{\infty}[M_{l^+}-E_{l^+}-M_{l^-}-E_{l^-}]P_l''(\cos\theta).$$

These are given in reference 7.

Using Eq. (8) and formulas of the type

$$u(p)=\frac{\gamma\cdot p+m}{[2m(E+m)]^{1/2}}\begin{pmatrix}\chi\\0\end{pmatrix}, \tag{12}$$

we may reduce the $\mathfrak{M}_i$'s to Pauli matrices and get

$$\mathfrak{F}_1=\frac{W-m_N}{8\pi W}[(E_1+m_N)(E_2+m_\Lambda)]^{1/2}\left[A_1+(W-m_N)A_4-\frac{t-m_K^2}{2(W-m_N)}(A_3-A_4)\right],$$

$$\mathfrak{F}_2=\frac{W-m_N}{8\pi W}\left(\frac{E_1+m_N}{E_2+m_\Lambda}\right)^{1/2}q\left[-A_1+(W+m_N)A_4-\frac{t-m_K^2}{2(W+m_N)}(A_3-A_4)\right],$$

$$\mathfrak{F}_3=\frac{W-m_N}{8\pi W}[(E_1+m_N)(E_2+m_\Lambda)]^{1/2}q[(W-m_N)A_2+(A_3-A_4)], \tag{13}$$

$$\mathfrak{F}_5=\frac{W-m_N}{8\pi W}\left(\frac{E_1+m_N}{E_2+m_\Lambda}\right)^{1/2}q^2[-(W+m_N)A_2+(A_3-A_4)].$$

III. LOW-ENERGY APPROXIMATION

The singularities of the S matrix come from the three Cutkosky diagrams[10] shown in Fig. 2(a), (b), (c). They represent the s-channel, t-channel, and u-channel singularities, respectively. The locations of these singularities can be seen from Fig. 3, where the cross-hatched region in the s-channel physical region corresponds to incident laboratory photon energy $(=E_\gamma)$ less than 1080 MeV.

Thus it is clear that close to the threshold of K^+-meson production, the t-channel singularities are considerably closer to the physical region than the u-channel singularities.[11] Another way of looking at this is to compare the location of the pole terms due to K^+ and Λ^0 exchange in the $\cos\theta$ plane. The K^+ exchange gives a pole at $\cos\theta=\omega/q$, whereas the Λ^0-exchange pole occurs at $\cos\theta=-E_2/q$. In Table I we compare them for various energies.

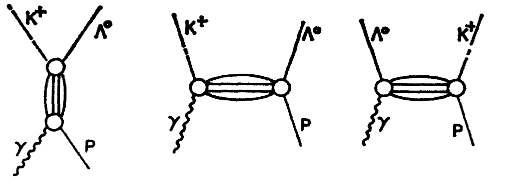

FIG. 2. (a), (b), and (c) The three Cutkosky diagrams in our reaction.

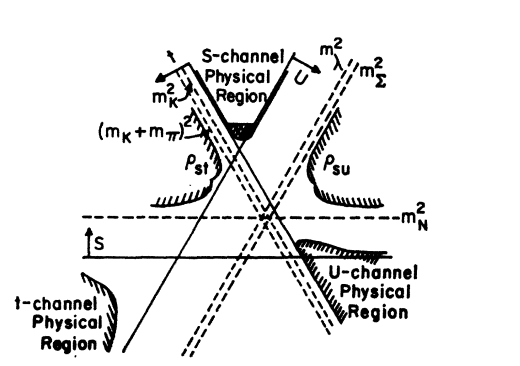

FIG. 3. "Mandelstam diagram" showing the location of the S-matrix singularities.

[10] R. E. Cutkosky, J. Math. Phys. 1, 429 (1960); also Phys. Rev. Letters 4, 624 (1960).

[11] See Kibble, Phys. Rev. 117, 1159 (1960) for the kinematics of Mandelstam representation.

We believe, therefore, that u-channel singularities have little influence upon low-energy K^+-meson photoproduction, and shall neglect the u-channel singularities altogether.[12,13] How good this approximation is, we do not know. But it may at least serve as a first try on the complicated problem. To include such terms would in any event require knowledge of the hyperon magnetic and transition moments.[14]

In the t channel we shall take the K^+ as well as the K^*-exchange[15] contributions. It is not known to date whether K^* has $J=0$ or 1.[16] K^* has recently been determined to be a vector meson. [See W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).] We note, however, that if K^* has $J=0$, there is no one-K^* exchange because the vertex $K^*-K-\gamma$ would be forbidden by virtue of the conservation of angular momenta and of parity. As will be seen later, K^* is important in our analysis. Thus we make the tentative conclusion that K^* is a "vector meson."

In the s channel we assume the dominant contribution to be the one-nucleon Born term and the second and third resonances of the πN system. (The 3–3 resonance

TABLE I. Location of the K^+ and Λ^0 exchange poles in the $\cos\theta$ plane for different incident photon energies E_γ.

E_γ (MeV)	950	1000	1050	1100	1150	1200
$\cos\theta_{K^+}$	4.1	2.8	2.3	2.0	1.9	1.76
$\cos\theta_{\Lambda^0}$	−9.0	−6.0	−4.8	−4.16	−3.7	−3.6

does not enter since it has $T=\frac{3}{2}$.) Further, a peak has been established in the reaction $\pi^-+p\rightarrow\Lambda^0+K^0$.[17] This was assigned the quantum numbers $T=\frac{1}{2}$, $J=\frac{1}{2}$ or $\frac{3}{2}$.[18] As the threshold behavior goes like q^{2L+1}, where

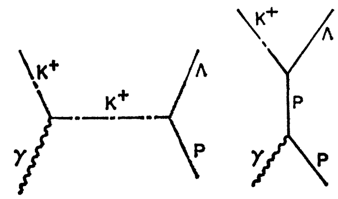

FIG. 4. (a), (b) The two Born terms used in our analysis.

L = angular momentum of final state, we believe that at "low energies" only S and P waves are excited to an appreciable degree and hence shall neglect the $D_{3/2}$ and $F_{5/2}$ πN resonances.[19] A resonance term for the Λ^0K^+ peak is retained and will be taken as $P_{1/2}$ or $P_{3/2}$. It corresponds to magnetic dipole (M_{1^-}) in the first case, and to magnetic dipole (M_{1^+}) or electric quadrupole (E_{1^+}) in the second case. Note that whether this is a resonance in the πN or $K\Lambda$ system cannot be determined from the production process alone.

IV. BORN TERMS AND RESONANCE CONTRIBUTIONS

The Born terms as given in Figs. 4(a) and 4(b) can be easily calculated to give, respectively,

$$-\frac{g_\Lambda e}{t-m_K^2}\frac{2}{s-m^2}\mathfrak{M}_2, \tag{14}$$

and

$$\frac{g_\Lambda}{s-m_N^2}[-(\mathfrak{M}_3+\mathfrak{M}_4)\mu_p]+\frac{g_\Lambda e}{s-m_N^2}\mathfrak{M}_1, \tag{15}$$

where g_Λ ($\equiv g_{\Lambda NK}$) is the renormalized and rationalized coupling constant, $\mu_p=1.8e/2m_N$ being the anomalous magnetic moment of the proton.

K^* enters into our reaction through the diagram shown in Fig. 5.

Now the vertex $K^*-K-\gamma$ is of the form[20]

$$\langle k;q|j_{K^*}{}^{\mu}|0\rangle=\frac{F[(q-k)^2]}{(4q_0k_0)^{1/2}}e^{\mu\nu\rho\sigma}\epsilon_\nu k_\rho q_\sigma, \tag{16}$$

where $j_{K^*}{}^{\mu}$ is the current 4-vector of the K^* vector meson field, and F stands for the form factor (unnormalized).

The vertex $K^*-p-\Lambda^0$, in analogy with $\gamma-p-\Lambda^0$,

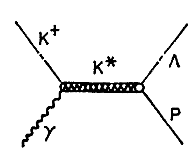

FIG. 5. The K^* exchange diagram.

[12] u-channel singularities might be important at high energies (incident photon energy $E_\gamma\approx 2$ BeV) as discussed by R. H. Capps, Phys. Rev. **126**, 324 (1962).

[13] There is also the possible cancellation of Σ^0 and Λ^0 exchange poles if $g_{\Sigma^0pK}\cong+g_{\Lambda^0pK}$. This is because the magnetic moment of $\Lambda^0(\mu_\Lambda)$ is $\cong-1.5\pm0.5M_B$ [R. L. Cool, E. Jenkins, T. F. Kycia, D. A. Hill, L. Marshall, and R. A. Schluter, Phys. Rev. **127**, 2223 (1962)]. However, the Argonne Lab group [W. Kernan, T. B. Novey, S. D. Warshae, and A. Wattenburg has reported the value $\mu_\Lambda=0.0\pm0.5M_B$ in the *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962)] whereas the $\Sigma^0-\Lambda^0$ transition moment $\mu_T\cong+0.6\mu_p$ (see reference 30), and they tend to cancel.

[14] The Λ^0-exchange pole is proportional to μ_Λ. If we take $\mu_\Lambda=-1M_B$, then by itself its contribution to $(d\sigma/d\Omega)$ is $(0.2-0.08\cos\theta)\times10^{-31}$ cm² at $E_\gamma=1054$ MeV.

[15] M. Alston, L. Alvarez, P. Eberhard, M. Good, W. Graziano, H. Ticho, S. Wojcicki, Phys. Rev. Letters **6**, 300 (1961). Also: G. Alexander, G. Kalbfleisch, D. Miller, and G. Smith, *ibid.* **8**, 447 (1962).

[16] See Chia-hwa Chan, Phys. Rev. Letters **66**, 383 (191), who favors $J=1$, and M. Alston, G. Kalbfleisch, H. Ticho, and S. Wojcicki [University of California Radiation Laboratory Report UCRL-10232 (to be published)], who favor $J=0$.

[17] L. Bertanza, P. L. Connolly, B. B. Culurick, F. R. Eisler, T. Morris, R. Palmer, A. Prodell, and N. P. Samios, Phys. Rev. Letters **8**, 332 (1962). References to earlier literatures can be found in this letter.

[18] Akira Kanazawa, Phys. Rev. **123**, 997 (1961).

[19] Cf., however, M. Gourdin and M. Rimpault, Nuovo Cimento **24**, 414 (1962), who had a quite different viewpoint.

[20] See, e.g., M. Gell'mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

has the following form:

$$\langle p_1; p_2|j_{K^*}{}^{\mu}|0\rangle=\left(\frac{m_N m_\Lambda}{E_1E_2}\right)^{1/2}\bar{u}(p_2)[G_1\gamma^\mu+iG_2\sigma^{\mu\nu}(p_2-p_1)_\nu+G_3(p_2-p_1)^\mu]u(p_1). \quad (17)$$

This is a consequence of Lorentz invariance and the fact that all the Dirac particles are on the mass shell. Conservation of current implies

$$\partial_\mu j^\mu=0, \quad (18)$$

or, in momentum space,

$$G_1\gamma^\mu(p_2-p_1)_\mu+G_3(p_2-p_1)^2=0, \quad (19)$$

i.e.,

$$G_3=-\Delta G_1/t, \quad (20)$$

where

$$\Delta=(m_\Lambda-m_N). \quad (21)$$

The propagator of the vector meson is

$$\frac{\delta_{\mu\nu}-(p_2-p_1)_\mu(p_2-p_1)_\nu/m_{K^*}{}^2}{t-m_{K^*}{}^2}, \quad (22)$$

where μ and ν are the polarization index of the emitted and absorbed vector mesons, respectively. From the complete antisymmetry of $e^{\mu\nu\rho\sigma}$, and the relation $p_2-p_1=k-q$, it follows immediately that G_3 does not contribute. Also the second term in (22) does not contribute. Thus, combining (16), (17), and (22), we have the total K^* contribution:

$$\frac{1}{t-m_{K^*}{}^2}\bar{u}(p_2)[C_1'e^{\mu\nu\rho\sigma}\gamma_\mu q_\nu k_\rho\epsilon_\sigma+iC_2'e^{\mu\nu\rho\sigma}q_\nu k_\rho\epsilon_\sigma\sigma_{\mu\mu'}(p_2-p_1)^{\mu'}]u(p_1), \quad (23)$$

where C_i' are proportional to G_iF.

Using the identity $\gamma_5=(1/4!)e_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$, and after a somewhat lengthy algebraic manipulation of the γ matrices, we can write (23) in terms of the $\mathfrak{M}_i$'s defined in (7):

$$\frac{1}{t-m_{K^*}{}^2}\bar{u}(p_2)[C_1'(\Delta\mathfrak{M}_1-\mathfrak{M}_4)+C_2'(t\mathfrak{M}_1-\mathfrak{M}_2+\Delta\mathfrak{M}_3)]u(p_1), \quad (24)$$

Δ being defined in (21). C_1' and C_2' are the analogs of charge and magnetic moments in electromagnetic interactions. In the Appendix we see how the ratio C_1'/C_2' can be related to various coupling constants involving K mesons. Combining (14), (15), and (24), we have the $\mathfrak{F}_i{}^0$ amplitudes

$$\mathfrak{F}_1{}^0=\frac{W-m_N}{2W}[(E_1+m_N)(E_2+m_\Lambda)]^{1/2}\frac{g_\Lambda e}{4\pi}\left\{\frac{1}{s-m_N}[1-(W-m_N)\mu_p/e]+\frac{1}{t-m_{K^*}{}^2}\left[\Delta C_1-(W-m_N)C_1+C_2t-\frac{t-m_K{}^2}{2(W-m_N)}(\Delta C_2+C_1)\right]\right\}, \quad (25)$$

$$\mathfrak{F}_2{}^0=\frac{W-m_N}{2W}\left(\frac{E_1+m_N}{E_2+m_\Lambda}\right)^{1/2}q\frac{g_\Lambda e}{4\pi}\left\{\frac{1}{s-m_N{}^2}\left[-1-(W+m_N)\frac{\mu_p}{e}\right]+\frac{1}{t-m_{K^*}{}^2}\left[-(\Delta C_1+tC_2)+(W+m_N)(-C_1)-\frac{t-m_K{}^2}{2(W+m_N)}(\Delta C_2+C_1)\right]\right\}, \quad (26)$$

$$\mathfrak{F}_3{}^0=\frac{W-m_N}{2W}[(E_1+m_N)(E_2+m_\Lambda)]^{1/2}q\frac{g_\Lambda e}{4\pi}\left\{\frac{1}{s-m_N{}^2}\frac{W-m_N}{k(\omega-q\cos\theta)}+\frac{1}{t-m_{K^*}{}^2}[(W-m_N)(-C_2)+(\Delta C_2+C_1)]\right\}, \quad (27)$$

$$\mathfrak{F}_4{}^0=\frac{W-m_N}{2W}\left(\frac{E_1+m_N}{E_2+m_\Lambda}\right)^{1/2}q^2\frac{g_\Lambda e}{4\pi}\left\{-\frac{1}{s-m_N{}^2}\frac{W+m_N}{k(\omega-q\cos\theta)}+\frac{1}{t-m_{K^*}{}^2}[(W+m_N)C_2+\Delta C_2+C_1]\right\}, \quad (28)$$

where C_1 and C_2 differ from C_1' and C_2' by certain constant factors.

The resonance state discussed in Sec. III will be assumed to be of the Breit-Wigner form:

$$q\frac{(kF)}{(s-s_0)+i\Gamma/2}. \quad (29)$$

This holds for both a $P_{1/2}$ and a $P_{3/2}$ resonance in the final state.

For a $P_{1/2}$ resonance, $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_4$ are given by the corresponding $\mathfrak{F}_i{}^0$'s, but $\mathfrak{F}_2$ becomes

$$\mathfrak{F}_2=\mathfrak{F}_2{}^0+q\frac{kF_1}{(s-s_0)+i\Gamma/2}. \quad (30)$$

For a $P_{3/2}$ resonance, either M_{1^+} or E_{1^+} can be assumed to take the form (30). In the absence of any information as to their relative magnitudes,[21] we shall

[21] In the case of the 3–3 resonance, $|M_{1^+}|\gg|E_{1^+}|$; for the second πN resonance, $|E_{2^-}|\gg|M_{2^-}|$.

only discuss two simple cases: (a) $|M_{1^+}|\ll|E_{1^+}|$ and (b) $|E_{1^+}|\ll|M_{1^+}|$.

In case (a), we have

$$\begin{aligned}\mathfrak{F}_1&=\mathfrak{F}_1^0+3\cos\theta q\frac{kF_2}{(s-s_0)+i\Gamma/2},\\ \mathfrak{F}_2&=\mathfrak{F}_2^0+2q\frac{kF_2}{(s-s_0)+i\Gamma/2},\\ \mathfrak{F}_3&=\mathfrak{F}_3^0-3q\frac{kF_2}{(s-s_0)+i\Gamma/2},\\ \mathfrak{F}_4&=\mathfrak{F}_4^0.\end{aligned}\tag{31}$$

In case (b), we have

$$\begin{aligned}\mathfrak{F}_1&=\mathfrak{F}_1^0+3\cos\theta q\frac{kF_3}{(s-s_0)+i\Gamma/2},\\ \mathfrak{F}_2&=\mathfrak{F}_2^0,\\ \mathfrak{F}_3&=\mathfrak{F}_3^0+3q\frac{kF_3}{(s-s_0)+i\Gamma/2},\\ \mathfrak{F}_4&=\mathfrak{F}_4^0.\end{aligned}\tag{32}$$

In terms of the $\mathfrak{F}_i$ amplitudes, the differential cross section for an unpolarized proton is

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}\{|\mathfrak{F}_1|^2+|\mathfrak{F}_2|^2+(|\mathfrak{F}_3|^2+|\mathfrak{F}_4|^2)(\hat{q}\cdot\hat{\epsilon})^2-2\,\mathrm{Re}(\mathfrak{F}_1{}^*\mathfrak{F}_2)(\hat{q}\cdot\hat{k})$$
$$+2\,\mathrm{Re}(\mathfrak{F}_1{}^*\mathfrak{F}_4)(\hat{q}\cdot\hat{\epsilon})^2+2\,\mathrm{Re}(\mathfrak{F}_2{}^*\mathfrak{F}_3)(\hat{q}\cdot\hat{\epsilon})^2+2\,\mathrm{Re}(\mathfrak{F}_3{}^*\mathfrak{F}_4)(\hat{k}\cdot\hat{q})(\hat{q}\cdot\hat{\epsilon})^2\},\tag{33}$$

or, averaging over photon polarization:

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}\{|\mathfrak{F}_1|^2+|\mathfrak{F}_2|^2+[\tfrac{1}{2}|\mathfrak{F}_3|^2+\tfrac{1}{2}|\mathfrak{F}_4|^2+\mathrm{Re}(\mathfrak{F}_1{}^*\mathfrak{F}_4)+\mathrm{Re}(\mathfrak{F}_2{}^*\mathfrak{F}_3)]\sin^2\theta$$
$$+\mathrm{Re}(\mathfrak{F}_3{}^*\mathfrak{F}_4-2\mathfrak{F}_1{}^*\mathfrak{F}_2)\cos\theta-\mathrm{Re}(\mathfrak{F}_3{}^*\mathfrak{F}_4)\cos^3\theta\}.\tag{34}$$

The polarization of Λ^0 along $\hat{k}\times\hat{q}\equiv\hat{n}$ is (for an unpolarized proton)

$$\frac{k}{q}\left(\frac{d\sigma}{d\Omega}\right)P_n=\tfrac{1}{2}\,\mathrm{Sp}(\mathfrak{F}\mathfrak{F}^+\sigma_n)$$
$$=2\frac{(\hat{q}\cdot\hat{\epsilon})^2}{|\hat{k}\times\hat{q}|}\,\mathrm{Im}\left[\frac{|\hat{k}\times\hat{q}|^2}{(\hat{q}\cdot\hat{\epsilon})^2}\mathfrak{F}_1\mathfrak{F}_2{}^*+(\hat{q}\cdot\hat{k})\mathfrak{F}_1\mathfrak{F}_4{}^*+\mathfrak{F}_1\mathfrak{F}_3{}^*-(\hat{q}\cdot\hat{k})\mathfrak{F}_2\mathfrak{F}_3{}^*-\mathfrak{F}_2\mathfrak{F}_4{}^*-(\mathfrak{F}_3\mathfrak{F}_4{}^*)(\hat{k}\times\hat{q})^2\right].\tag{35}$$

For an unpolarized photon, it is

$$\frac{k}{q}\left(\frac{d\sigma}{d\Omega}\right)P_n=\sin\theta\,\mathrm{Im}\{2(\mathfrak{F}_1\mathfrak{F}_2{}^*)+(\mathfrak{F}_1\mathfrak{F}_3{}^*)+\cos\theta(\mathfrak{F}_1\mathfrak{F}_4{}^*-\mathfrak{F}_2\mathfrak{F}_3{}^*)-(\mathfrak{F}_2\mathfrak{F}_4{}^*)-(1-\cos^2\theta)(\mathfrak{F}_3\mathfrak{F}_4{}^*)\}.\tag{36}$$

V. NUMERICAL RESULTS

We now compare our formulas with experimental data. To do this we note that the various constants appearing in (25) to (32) are to be regarded as parameters. More precisely, in our model there are altogether six unknowns: g_Λ, C_1, C_2, F_i, S_0, and Γ. S_0 and Γ, however, can be approximately determined from the experiment $\pi^-+p\rightarrow K^0+\Lambda^0$. We choose $S_0=2.88\times10^6$ (MeV)^2 and $\Gamma/2=8\times10^4$ MeV, corresponding to total energy $W=1700$ MeV and full width of the resonance 60 MeV.

In order to determine the other four constants, we have substituted the $\mathfrak{F}_i^0$ amplitudes into (34) for $d\sigma/d\Omega$, which is then written in the form $a+b\cos\theta+c\cos^2\theta$, the three coefficients being polynomials of C_1, C_2, and g_Λ. As functions of C_1 and C_2, b turns out to be a hyperbola and c an ellipse. If we further require that b should be positive and comparable to a, and c should not be negative and large, we found that C_1 and C_2 are centered around $C_1\cong1.5\times10^{-3}$ (MeV)$^{-1}$ and $C_2\cong0.5\times10^{-6}$ (MeV)$^{-2}$. After this we put in the resonance state, a readjustment of C_1 and C_2 were made to yield a "good" fit to the data. The coupling constant $g_\Lambda^2/4\pi$ is mainly fixed by low-energy data outside the region of the resonance. The results of all these are listed in Table II. It goes without saying that small changes applied to the parameters simultaneously might lead us back to a similar final result. But it seems that if any one of the parameters undergoes a drastic change, a reasonable fit is unlikely to be found.

We summarize our results in Table III, in which we give $(d\sigma/d\Omega)_0$, which comes from $\mathfrak{F}_i^0$ alone, as well as $(d\sigma/d\Omega)_R$, which gives the various resonance contributions.

The comparison with experiments are presented in Figs. 6 through 8. The first four deal with angular

TABLE II. Parameters used in the model.

$g_\Lambda^2/4\pi$	4.0	$\Gamma/2$ (MeV^2)	8×10^4
C_1 (MeV^{-1})	1.30×10^{-3}	$F_1(M_{1^-})$ (MeV^{-1})	5.56×10^{-6}
C_2 (MeV^{-2})	0.40×10^{-6}	$F_2(M_{1^+})$ (MeV^{-1})	-3.52×10^{-6}
s_0 (MeV^2)	2.88×10^6	$F_3(E_{1^+})$ (MeV^{-1})	-2.62×10^{-6}

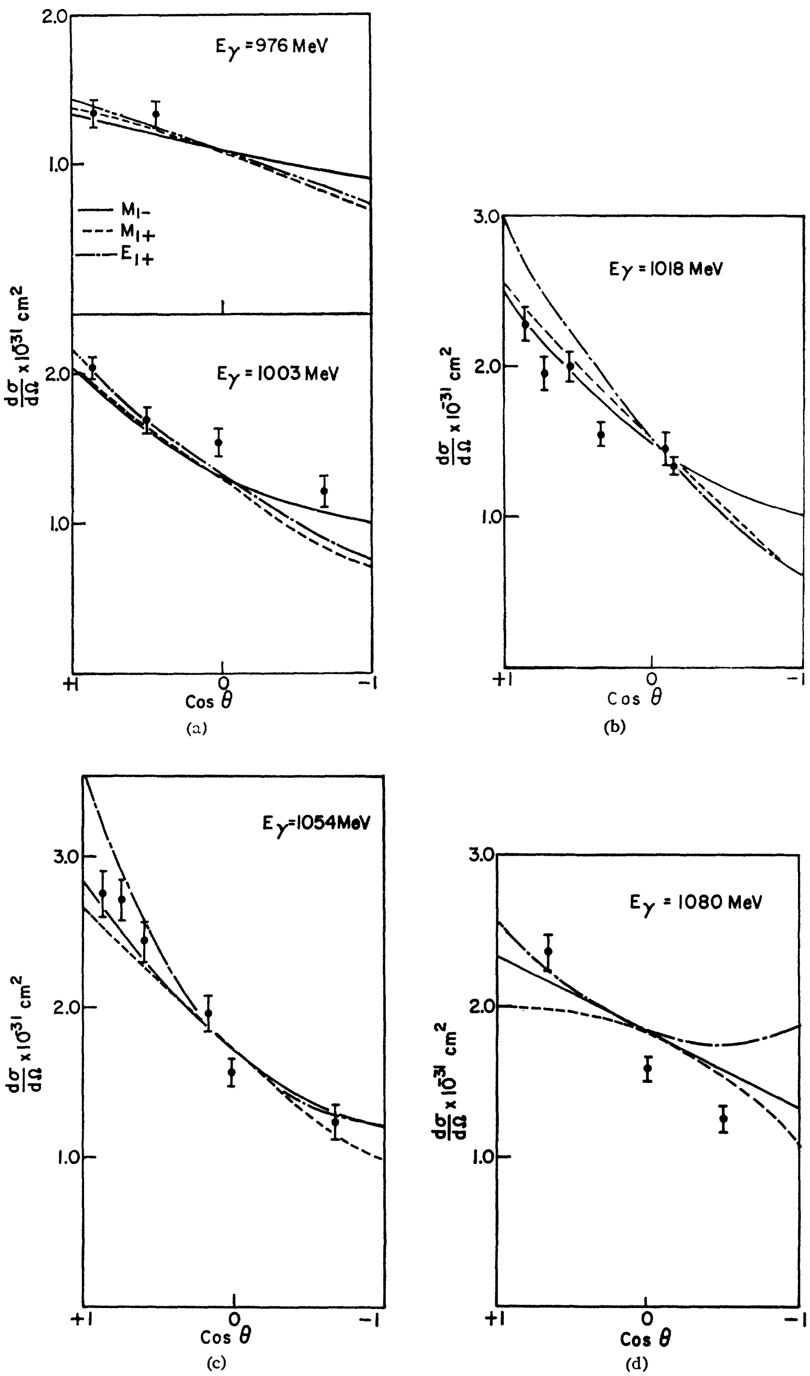

FIG. 6. (a)–(d) Differential cross sections for photoproduction. The data are those of reference 1. The solid curves denote a $P_{1/2}(M_1^-)$ resonant state, while the broken and the dot-dash curves correspond to a M_1^+ and E_1^+ resonance, respectively.

distributions at photon incident energies $E_\gamma=976$, 1003, 1018, 1054, and 1080. The fifth is the excitation function which gives $(d\sigma/d\Omega)_{\theta=\pi/2}$ against the total c.m. energy of the system. In Fig. 8 we have plotted the polarization of the produced Λ^0 particle at $E_\gamma=1054$. There is one preliminary experimental point at $\theta=80°$ at this energy which is $P_n=0.40\pm0.12$,[22] ($|\alpha|=0.61$).

Notice that $\cos^3\theta$ terms in $d\sigma/d\Omega$ are not included. They turn out to be small throughout our energy range.

Before we go on, let us make a few remarks about the relative strength of the various terms in this model. In accordance with the choice of Table II, the Born terms and the K^*-exchange contributions are about the same. As these terms are intermingled in a rather

[22] B. D. McDaniel, R. L. Anderson, E. Gabathuler, D. P. Jones, A. J. Sadoff, and H. Thom, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 266.

TABLE III. Differential cross sections (in units of 10^{-31} cm^2) resulting from our model for different incident photon energies. The subscripts 0, 1, 2, 3 denote no resonance, M_1^-, M_1^+, and E_1^+ in resonance, respectively.

E_γ (MeV)	976	1003	1018	1054	1080
$(d\sigma/d\Omega)_0$	$1.05-0.03\cos\theta$ $-0.05\cos^2\theta$	$1.23+0.14\cos\theta$ $-0.01\cos^2\theta$	$1.34+0.35\cos\theta$ $+0$	$1.36+0.58\cos\theta$ $+0.12\cos^2\theta$	$1.41+0.63\cos\theta$ $+0.07\cos^2\theta$
$(d\sigma/d\Omega)_1$	$0.03+0.24\cos\theta$ $+0.08\cos^2\theta$	$0.07+0.37\cos\theta$ $+0.21\cos^2\theta$	$0.14+0.41\cos\theta$ $+0.28\cos^2\theta$	$0.35+0.25\cos\theta$ $+0.19\cos^2\theta$	$0.41-0.11\cos\theta$ $-0.09\cos^2\theta$
$(d\sigma/d\Omega)_2$	$0.03+0.38\cos\theta$ $+0$	$0.07+0.51\cos\theta$ $+0.08\cos^2\theta$	$0.14+0.60\cos\theta$ $+0.10\cos^2\theta$	$0.35+0.32\cos\theta$ $-0.06\cos^2\theta$	$0.41-0.18\cos\theta$ $-0.34\cos^2\theta$
$(d\sigma/d\Omega)_3$	$0.03+0.41\cos\theta$ $+0.03\cos^2\theta$	$0.07+0.57\cos\theta$ $+0.15\cos^2\theta$	$0.14+0.82\cos\theta$ $+0.31\cos^2\theta$	$0.35+0.51\cos\theta$ $+0.47\cos^2\theta$	$0.41-0.29\cos\theta$ $+0.32\cos^2\theta$

complicated fashion, it is not a simple matter to single out the contribution of any one of them and subject it to an experimental test. (See, however, the discussion in Sec. VI.) The resonance state does not contribute much (about 25%) to the excitation curve, or $(d\sigma/d\Omega)_{\theta=\pi/2}$, but it accounts for a large part of the forward K^+-meson peak, especially for photon energies around 1000 MeV. It is also indispensable for the polarization. For without the resonant state the other terms do not give rise to any polarization of the produced Λ^0.

VI. DISCUSSION

The fits with data in the previous section indicate that an M_1^- resonant state is better than the other two. A resonant M_1^+ state is characterized by a very small $\cos^2\theta$ term, while an E_1^+ resonance gives a large interference $\cos\theta$ term. If we take the single polarization measurement in Fig. 11 seriously, then an E_1^+ resonance is actually excluded. Whether the M_1^- resonant state (or $P_{1/2}$ in the final state) might be a Ball-Frazer[23] type resonance occurring at the ΣK threshold can only be answered after the coupled channel problem has been solved. It is also to be noted that in the reaction $\pi^-+p\to\Lambda^0+K^0$, the angular distribution and the total cross section, as analyzed by Feld and Layson,[24] are very suggestive of a $P_{1/2}$ resonant state of the πp or ΛK system.

The coupling constant $g_{\Lambda NK}^2/4\pi$ is found to be 4.0. This is in agreement with several authors.[25] It is smaller than $g_{NN\pi}^2/4\pi$, but still of the same order. Thus the hope that perturbation expansion might be useful in strange-particle physics is waning, as has been emphasized by Dalitz and Tuan.[26]

Finally, we note that the reaction $\gamma+n\to K^0+\Lambda^0$ is very closely related to $\gamma+p\to K^++\Lambda^0$. Indeed, according to our model the $(K^0\Lambda^0)$ and $(K^+\Lambda^0)$ resonant states are a pair of isospin doublets, and so are $(K^*)^-$ and $(K^*)^0$. The "coupling constants" involved are the same, provided that only the isoscalar part of the photon interactions is important. In any case the "coupling constants" involved would not be drastically different. The only other differences are:

(1) In the one nucleon Born term there is no charge interaction; also the neutron magnetic moment μ_n is related to that of the proton by $\mu_n\cong-\mu_p$.

(2) The K^0 exchange does not contribute since we are dealing with a real photon in the vertex $K^0-K^0-\gamma$.[27] Thus the reaction $\gamma+n\to K^0+\Lambda^0$ is actually to a certain extent determined by $\gamma+p\to K^++\Lambda^0$. Measurements on $\gamma+n\to K^0+\Lambda^0$ would, therefore, serve to clarify the photoproduction of K^+ mesons and Λ^0 particles and in particular would test the model proposed here.

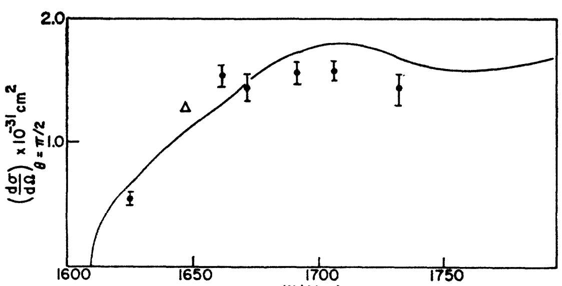

FIG. 7. $(d\sigma/d\Omega)_{\theta=\pi/2}$ plotted vs the total c.m. energy. The data are from reference 1. (Δ is obtained as an extrapolation from $(d\sigma/d\Omega)$ at $E_\gamma=976$ MeV.)

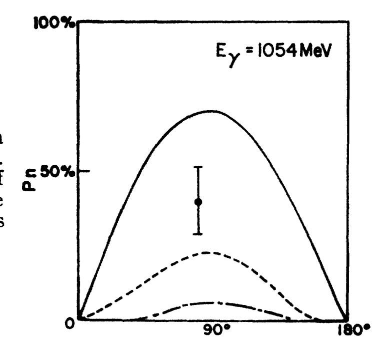

FIG. 8. Polarization of the Λ^0 at $E_\gamma=1054$. The notation is that of Figs. 6 (a)–(d). The experimental point is from reference 22.

[23] J. Ball and W. Frazer, Phys. Rev. Letters **7**, 204 (1961).
[24] B. T. Feld and W. Layson, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 147.
[25] R. H. Capps, Phys. Rev. **121**, 291 (1961). See also reference 18.
[26] R. Dalitz and S. Tuan, Ann. Phys. (N. Y.) **3**, 307 (1960).
[27] G. Feinberg, Phys. Rev. **109**, 1381 (1958).

ACKNOWLEDGMENTS

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APPENDIX

In this Appendix we treat the coupling $K^*-\Lambda-N$. Thus the reaction under consideration is $\pi+\bar{K}\rightarrow\Lambda+\bar{p}$. Define the 4-momenta as in Fig. 9. The invariant variables are

$$t=(k+q)^2,\quad s=(\bar{p}-q)^2,\quad \bar{s}=(\bar{p}-k)^2. \tag{A1}$$

As usual, the S matrix is written as

$$S_{fi}=-\frac{i}{(2\pi)^2}\delta^4(k+q-p-\bar{p})\left(\frac{m_\Lambda m_p}{E_\Lambda E_p}\right)^{1/2}T_{fi}, \tag{A2}$$

where T_{fi} can be expressed in the form

$$T_{fi}=\bar{u}_\Lambda[A+\tfrac{1}{2}\gamma\cdot(k-q)B]v_{\bar{p}}, \tag{A3}$$

$v_{\bar{p}}$ being the antiparticle spinor; the metric is defined in footnote 8. The differential cross section can be written as

$$\frac{d\sigma}{d\Omega}=\frac{p}{k}\left|\frac{(m_\Lambda m_p)^{1/2}}{4\pi W}T_{fi}\right|^2, \tag{A4}$$

and T_{fi} reduces to Pauli spinors of the form

$$T_{fi}=\chi_\Lambda^+(h_1\boldsymbol{\sigma}\cdot\mathbf{p}_\Lambda+h_2\boldsymbol{\sigma}\cdot\mathbf{k})\chi_{\bar{p}} \tag{A5}$$

in the c.m. system, in an obvious notation. Then the relation between A, B, and h_1 and h_2 is given by

$$h_1=\frac{1}{[4m_\Lambda m_p(E_\Lambda+m_\Lambda)(E_p+m_p)]^{1/2}}\{\tfrac{1}{2}(k_0-q_0)[(E_\Lambda+m_\Lambda)-(E_p+m_p)]B \\ -[(E_\Lambda+m_\Lambda)+(E_p+m_p)]A+2\mathbf{p}_\Lambda\cdot\mathbf{k}B\}, \tag{A6}$$

$$h_2=\frac{-1}{[4m_\Lambda m_p(E_\Lambda+m_\Lambda)(E_p+m_p)]^{1/2}}[(E_\Lambda+m_\Lambda)(E_p+m_p)B+p^2B]. \tag{A7}$$

(A6) and (A7) are generalizations of (3.3) and (3.4) of F. F.[28]

Now K^* is supposed to be a strong p-wave resonance in the $\pi\bar{K}$ system. Following F. F., we shall use helicity amplitudes defined as[29]

$$\langle\bar{K}\pi|T^{J=1,T=\frac{1}{2}}|\Lambda(+)\bar{p}(+)\rangle=\mathcal{P}_{T=\frac{1}{2}}4\pi\frac{1}{(4E_\Lambda E_p q_0 k_0)^{1/2}}\frac{3}{2}T_+{}^1\cos\theta, \tag{A8}$$

$$\langle\bar{K}\pi|T^{J=1,T=\frac{1}{2}}|\Lambda(+)\bar{p}(-)\rangle=-\mathcal{P}_{T=\frac{1}{2}}4\pi\frac{E_\Lambda+E_p}{(4E_\Lambda E_p q_0 k_0)^{1/2}}\frac{3}{2}\frac{1}{\sqrt{2}}T_-{}^1\sin\theta e^{i\phi}, \tag{A9}$$

where $\mathcal{P}_{T=\frac{1}{2}}$ is the isospin projection operator for $T=\frac{1}{2}$, θ is the c.m. scattering angle, and ϕ is the azimuthal angle of $\mathbf{p}_\Lambda$. Evaluating $T_{fi}=\chi_\Lambda^+(h_1\boldsymbol{\sigma}\cdot\mathbf{p}_\Lambda+h_2\boldsymbol{\sigma}\cdot\mathbf{k})\chi_{\bar{p}}$ in the c.m. system in which

$$\hat{p}_\Lambda=\hat{e}_z,\quad \hat{k}=\hat{e}_z\cos\theta+\hat{e}_x\sin\theta, \tag{A10}$$

and comparing with (A8) and (A9), we get

$$T_+{}^1(t)=\frac{e^{i\alpha}(m_\Lambda m_p)^{1/2}}{4\pi t^{1/2}}\frac{k}{p}\int dx\,P_1(x)[p^2h_1(x)+pkxh_2(x)], \\ T_-{}^1(t)=\frac{e^{i\alpha}(m_\Lambda m_p)^{1/2}}{4\pi t^{1/2}}k^2\sqrt{2}\int dx\,h_2(x)[xP_1(x)-P_2(x)]. \tag{A11}$$

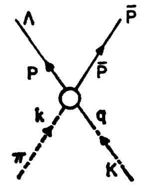
Taking $e^{i\alpha}=-1$, then [these formulas are generalizations of (II.11) of Dreitlein and Lee[30]]

$$T_+{}^1(t)=\frac{1}{8\pi t^{1/2}}\frac{(t^{1/2}+2M)(1-\Delta^2/t)}{[(E_\Lambda+m_\Lambda)(E_p+m_p)]^{1/2}}\frac{k}{p}\left\{\frac{p^2}{1-\Delta^2/t}A_1+pkM[\tfrac{2}{3}B_2+\tfrac{1}{2}B_0]-\frac{m\delta\Delta p^2}{t}\frac{1}{1-\Delta^2/t}B_1\right\}, \tag{A12}$$

$$T_+{}^1(t)=\frac{k^2}{8\pi}\frac{(t^{1/2}+2M)(1-\Delta^2/t)}{[(E_\Lambda+m_\Lambda)(E_p+m_p)]^{1/2}}\frac{1}{3\sqrt{2}}(B_0-B_2), \tag{A13}$$

[28] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1603 (1960).
[29] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[30] J. Dreitlein and B. W. Lee, Phys. Rev. **124**, 1274 (1961).

FIG. 9. Kinematics for the process $\pi+\bar{K}\to\Lambda+\bar{N}$.



where we defined

$$(A_J,B_J)=\int dx\,P_J(x)(A,B), \tag{A14}$$

and

$$\begin{aligned} m&=\tfrac{1}{2}(m_K+m_\pi), & \delta&=(m_K-m_\pi),\\ M&=\tfrac{1}{2}(m_\Lambda+m_p), & \Delta&=(m_\Lambda-m_p). \end{aligned} \tag{A15}$$

In terms of these definitions we get symmetrical expressions for the momentum and energy variables:

$$\begin{aligned} p^2&=(1-\Delta^2/t)(\tfrac{1}{4}t-M^2), & k^2&=(1-\delta^2/t)(\tfrac{1}{4}t-m^2),\\ E_\Lambda&=\tfrac{1}{2}t^{1/2}(1+2\Delta M/t), & E_p&=\tfrac{1}{2}t^{1/2}(1-2\Delta M/t),\\ q_0&=\tfrac{1}{2}t^{1/2}(1+2\delta m/t), & k_0&=\tfrac{1}{2}t^{1/2}(1-2\delta m/t). \end{aligned} \tag{A16}$$

Also, we have

$$\begin{aligned} s&=\left(-\frac{t}{2}+\frac{2}{t}M\Delta m\delta\right)+\tfrac{1}{2}\Sigma+2\cos\theta[(1-\Delta^2/t)\\ &\qquad\times(\tfrac{1}{4}t-M^2)(1-\delta^2/t)(\tfrac{1}{4}t-m^2)]^{1/2},\\ \bar{s}&=\left(-\frac{t}{2}-\frac{2}{t}M\Delta m\delta\right)+\tfrac{1}{2}\Sigma-2\cos\theta[(1-\Delta^2/t)\\ &\qquad\times(\tfrac{1}{4}t-M^2)(1-\delta^2/t)(\tfrac{1}{4}t-m^2)]^{1/2}, \end{aligned} \tag{A17}$$

where $\Sigma=\Sigma m_i^2=\Delta^2+4M^2+\delta^2+4m^2$. The "crossing symmetry" is expressed by the transformation

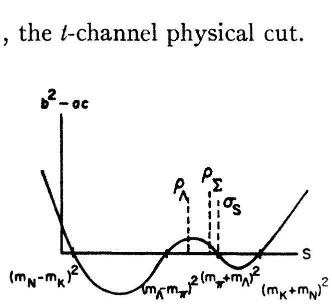
$$s\leftrightarrow\bar{s}\rightleftarrows\Delta\leftrightarrow-\Delta\quad\text{and}\quad\cos\theta\leftrightarrow-\cos\theta.$$

The partial-wave amplitude analyticity properties have been well discussed by Kennedy and Spearman.[31] Following their notation we give briefly the results for the singularities of our reaction in the t plane. Because of the complexity of the masses involved, we shall give numerical values whenever the complete expression gets messy. Thus we have:

(1) Cuts $-\infty$ to δ^2, $4m^2$ to ∞, to define s and $\bar{s}$ in the t plane.

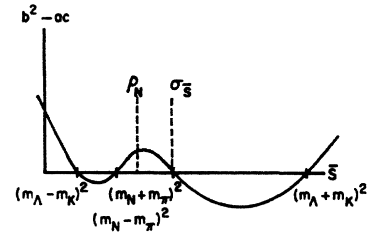
(2) A cut $(2M)^2$ to ∞, the t-channel physical cut.

FIG. 10. b^2-ac vs s, the notation is that of reference 31.



³¹ J. Kennedy and T. D. Spearman, Phys. Rev. **126**, 1596 (1962).

FIG. 11. b^2-ac vs $\bar{s}$, the notation is that of reference 31.



(3) A cut $-\infty$ to 0; a cut $-\infty$ to 0.03×10^6 (MeV)^2, for $s>(m_K+m_N)^2$ (see Fig. 10); for $\bar{s}>(m_\Lambda+m_K)^2$ (see Fig. 11), a cut from $-\infty$ to -0.27×10^6 (MeV)^2.

(4) Pole terms: (a) $s=m_\Lambda^2$, a cut 0.13 to 0.39×10^6 (MeV)^2; (b) $s=m_\Sigma^2$, a cut 0.12 to 0.28×10^6 (MeV)^2; (c) $\bar{s}=m_N^2$, a cut 0.12 to 0.40×10^6 (MeV)^2.

(5) Complex singularities arising from the ranges $(m_\Lambda+m_\pi)^2<s<(m_K+m_N)^2$ and $(m_N+m_\pi)^2<\bar{s}<(m_\Lambda+m_K)^2$ (see Figs. 10 and 11). These give rise to the curves shown in Fig. 12. (We have neglected two curves close to the origin.)

These results show that the "pole terms" give rise to singularities closest to the physical region. In the absence of detailed information, we shall take these terms to represent the s and $\bar{s}$ channel singularities. A simple calculation gives the contribution of these pole terms to the amplitudes A and B for isospin $\frac{1}{2}$.

(1) One nucleon pole in the $\bar{s}$ spectrum:

$$\begin{pmatrix}A\\B\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\Delta\\1\end{pmatrix}\frac{G_{NN\pi}g_{p\Lambda K}}{\bar{s}-m_N^2}\frac{1}{\sqrt{6}}. \tag{A18}$$

(2) Λ pole in s spectrum:

$$\begin{pmatrix}A\\B\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\Delta\\1\end{pmatrix}\frac{g_{\Lambda\Lambda\pi}g_{p\Lambda K}}{s-m_\Lambda^2}\frac{1}{\sqrt{3}}. \tag{A19}$$

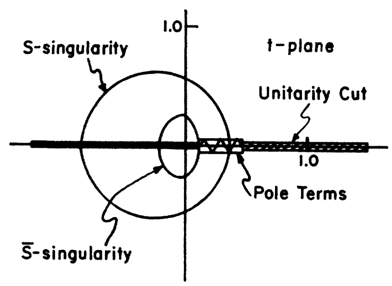
This vanishes approximately since $g_{\Lambda\Lambda\pi}=0$ if isospin is conserved.

(3) Σ pole in s spectrum:

$$\begin{pmatrix}A\\B\end{pmatrix}=-\begin{pmatrix}\frac{1}{2}(2m_\Sigma-m_\Lambda-m_N)\\1\end{pmatrix}\frac{g_{\Sigma\Lambda\pi}g_{\Sigma pK}}{s-m_\Sigma^2}\frac{1}{\sqrt{6}}. \tag{A20}$$

The coupling constants are so defined that the equiva-

FIG. 12. Partial-wave amplitude singularities in the t plane for $\pi+\bar{K}\to\Lambda+\bar{N}$.



lent interaction Hamiltonian can be written as

$$\mathcal{H}=G_{NN\pi}\bar{\Psi}_N\gamma_5\boldsymbol{\tau}_N\Psi_N\cdot\boldsymbol{\phi}_\pi+g_{\Sigma\Lambda\pi}\bar{\Psi}_\Sigma\gamma_5\Psi_\Lambda\cdot\boldsymbol{\phi}_\pi+g_{\Sigma NK}\bar{\Psi}_\Sigma\gamma_5\bar{\phi}_K\cdot\boldsymbol{\tau}\Psi_N+g_{\Lambda NK}\bar{\Psi}_\Lambda\gamma_5\bar{\phi}_K I\Psi_N+g_{\Lambda\Lambda\pi}\bar{\Psi}_\Lambda\gamma_5\Psi_\Lambda\phi_\pi+\text{other interaction terms.} \quad \text{(A21)}$$

Thus there are factors $\pm\sqrt{2}$, etc., involved in the Born terms. The factor $1/\sqrt{3}$ comes from Clebsch-Gordan coefficients.

Let us now consider the "form factors" of the $K^*-\Lambda-N$ coupling which are defined through the expression:

$$\langle 0|j_{K^*}{}^\mu|\Lambda\bar{N}\rangle=\left(\frac{m_\Lambda m_N}{E_\Lambda E_N}\right)^{1/2}\bar{u}(p_\Lambda)[\gamma^\mu F_1+iF_2\sigma^{\mu\nu}(p_\Lambda+\bar{p}_N)_\nu+F_3(p_\Lambda+\bar{p}_N)^\mu]v(\bar{p}_N), \quad \text{(A22)}$$

with the condition

$$F_1\cdot\Delta=-(p_\Lambda+\bar{p}_N)^2F_3.$$

The F_i's satisfy dispersion relations which we assume to be unsubtracted. The matrix element can be dispersed to give the absorptive part:

$$A^\mu=-8\pi^4\textstyle\sum_n\langle 0|j_{K^*}{}^\mu|n\rangle\langle n|f_N(0)|\Lambda\rangle\delta^4(p+\bar{p}-p_N)(E_\Lambda/m_\Lambda)^{1/2}. \quad \text{(A23)}$$

If we retain only a p-wave K^* intermediate state, and write the vertex $K^*-\bar{K}-\pi$ in the form:

$$\langle 0|j_{K^*}{}^\mu|\bar{K}\pi\rangle=\frac{i\gamma_{K^*\bar{K}\pi}}{(4q_0k_0)^{1/2}}F_{K^*\bar{K}\pi}{}^*(t)(q-k)^\mu\mathcal{P}_{T=\frac{1}{2}}, \quad \text{(A24)}$$

then we can put these together to arrive at

$$\text{Im}F_1=-\frac{(m_\Lambda+E_\Lambda)^{1/2}(m_N+E_N)^{1/2}}{(t^{1/2}+2M)}\gamma_{K^*\bar{K}\pi}F_{K^*\bar{K}\pi}{}^*(t)\frac{q}{p^2}\left[-MT_+{}^1+\frac{t^{1/2}}{2\sqrt{2}}T_-{}^1\right], \quad \text{(A25)}$$

$$\text{Im}F_2=\frac{1}{t^{1/2}}\frac{(m_\Lambda+E_\Lambda)^{1/2}(m_N+E_N)^{1/2}}{(t^{1/2}+2M)}\gamma_{K^*\bar{K}\pi}F_{K^*\bar{K}\pi}{}^*(t)\frac{q}{p^2}\left[\frac{t^{1/2}}{2}T_+{}^1+\frac{M}{\sqrt{2}}T_-{}^1\right]. \quad \text{(A26)}$$

We will not attempt to calculate F_1 and F_2. Rather we observe that since K^* is supposed to be a strong resonance, the spectral functions get the most contribution near this energy. We may then get an estimate of the ratio F_1/F_2 or C_1/C_2 or C_1'/C_2' (see Sec. IV) by calculating $\text{Im}F_1/\text{Im}F_2$ at $t=m_{K^*}{}^2$. $T_+{}^1$ and $T_-{}^1$ can be replaced by the pole terms as given in (A18) to (A20). Using (A13), (A14), (A15), (A25), and (A26), we get finally

$$\frac{F_1}{F_2}=2M\frac{0.21G_{NN\pi}g_{\Lambda pK}+0.29g_{\Lambda\Lambda\pi}g_{\Lambda pK}+0.25g_{\Sigma\Lambda\pi}g_{\Sigma pK}/\sqrt{2}}{0.04G_{NN\pi}g_{\Lambda pK}-0.09g_{\Lambda\Lambda\pi}g_{\Lambda pK}-0.79g_{\Sigma\Lambda\pi}g_{\Sigma pK}/\sqrt{2}}. \quad \text{(A27)}$$

Our result in the text, i.e., $C_1/C_2=(1.3/0.4)\times10^{-3}\approx 3M$, is reasonable according to this expression. Although no definite conclusion can be drawn from it since only $G_{NN\pi}$ is known.